Homework 5 – AS.171.627 – Zakamska

1. Aliens in distress (3+3+1 points)

(a) A space ship with volume V is filled with mono-atomic gas (particle mass m) at pressure P and temperature T_0. How many particles in the ship have vector velocities in the range between $\mathbf{v}$ to $\mathbf{v} + \mathrm{d}\mathbf{v}$? What about the number of particles with absolute values of velocity between v and $v + \mathrm{d}v$? How many have kinetic energies between E and $E + \mathrm{d}E$? What is the average kinetic energy of gas particles expressed in units of kT_0?

(b) The ship's hull develops a tiny hole (cross section S, such that $\sqrt{S} \ll$ mean free path of gas particles), and the ship starts slowly leaking gas into surrounding vacuum. How many particles are lost through the crack per unit time? What is the escaping particles' total kinetic energy (per unit time)? What is their average kinetic energy expressed in units of kT, where T is the instantaneous temperature inside the ship?

(c) Is the temperature inside the ship increasing, decreasing or staying constant? Why? (2 points extra credit: calculate temperature as a function of time.)

Hint: much of this problem is textbook material for upper-level statistical physics.

2. Virial theorem in spherical potentials (2 points). The virial theorem can be applied to test particles orbiting in fixed potentials as well as to N-body systems. Show that the mean-square velocity $\langle v^2 \rangle$ of a test particle orbiting in the potential of a point mass M satisfies $\langle v^2 \rangle = GM\langle 1/r \rangle$. What is the analogous expression for $\langle v^2 \rangle$ for a particle orbiting in a logarithmic potential, $\Phi(r) = V_c^2 \ln r$?

3. Stellar wind (3 points). A massive star produces a spherically symmetric outflow of ionized gas observed via optically thin emission lines. The velocity of the outflow is v_0 and the surface brightness of the emission is $\propto R^{-\alpha}$ at $R > R_0$, with $\alpha > 2$. What is the observed line-of-sight velocity dispersion of the emission lines as a function of R for $R > R_0$? What is the functional form of the line-of-sight velocity profile?

4. Local group timing (3 points). M31 (Andromeda) and the Galaxy are by far the two most luminous (and presumably most massive) galaxies in the Local Group. Remarkably, the two galaxies are moving towards each other, rather than participating in the Hubble expansion. In 1959 Kahn and Woltjer recognized that the most natural interpretation of this fact is that their expansion was slowed, halted and reversed by their gravitational attraction, and that this hypothesis provided a way to estimate their combined mass. Treating the galaxies as isolated point masses, assuming that their orbital angular momentum is negligible, and assuming that their separation was zero at the Big Bang, estimate (a) the total mass $M_{\mathrm{MW}} + M_{\mathrm{M31}}$ and (b) when the two galaxies will collide. Express your answer in terms of the age of the Universe, the current separation of the two galaxies d and their relative line of sight velocity $v_{\parallel}$. Then look up these values and provide numerical answers for (a) and (b) (don't forget to give references if appropriate).

5. Energy distribution, part 1 (3 points). Consider a spherical system with distribution function $f(E, L)$. Since this is a function of integrals of motion (E – orbital energy and L – orbital angular momentum), it automatically satisfies the Boltzmann equation. Let $N(E, L)dEdL$ be the number of stars in the range E to $E + dE$ and L to $L + dL$. Find $N(E, L)$ in terms of $f(E, L)$. Hint: your answer should involve the radial period $T_r(E, L)$.

6. Energy distribution, part 2 (2 points). Consider an equilibrium stellar system of N stars of mass m, where $N \gg 1$, there is no mass in non-stellar components, and the graviational potential is $\Phi(\mathbf{x})$. The individual energy of a star at phase-space position $(\mathbf{x}_i, \mathbf{v}_i)$ is $\varepsilon_i = m(v_i^2/2 + \Phi(\mathbf{x}_i))$. Let $E' = \sum_{i=1}^{N} \varepsilon_i$ be the sum of individual energies. What is the relation of E' to the total energy of the system?