Homework 2 – AS.171.627 – Zakamska

1. Bob (1 point). Bob, a high-school student working on an astronomy project, takes the map of the constellations printed in a popular astronomy book, looks up the stellar type of all the stars mentioned on it on Wikipedia and finds that there are as many red stars as there are blue stars. He knows that red stars have low mass and blue stars have high mass. He concludes that the rate at which stars form is independent of mass. List at least two problems with this argument.

2. Malmquist bias (3 points). (3.4 from BM) Evaluate the ratio of the mean luminosities of (a) a mag-limited sample and (b) a volume-limited sample of objects with luminosity function given by equation

$$\Phi(M) = \frac{\Phi_0}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{(M - M_0)^2}{2\sigma^2}\right). \tag{1}$$

Assume that the population is homogeneously distributed in space. If both surveys are infinitely deep, are the mean luminosities the same or different from one another?

3. Surface brightness and inclination (4 points). (1.4 of BT) An axisymmetric transparent galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ having axis ratio q. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes and central surface brightness I_n. A second distant observer, observing the galaxy from a line of sight that is inclined by an angle i to the symmetry axis, sees an image with elliptical isophotes with axis ratio $Q < 1$ and central surface brightness I_0.

(a) What is the relation between I_0, I_n and Q? Hint: the answers are different for oblate ($q < 1$) and prolate ($q > 1$) galaxies.

(b) What is the relation between q, Q and i?

(c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10 deg of the symmetry axis? From within 10 deg of the equatorial plane?

4. Potential and brightness profile of a globular cluster (2+3+2+1 points).

(a) (2.9 from BT) If a transparent, spherical stellar system has constant mass-to-light ratio γ, prove that the central potential is (Ciotti 1991)

$$\Phi(0) = -4G\gamma \int_0^\infty dR I(R), \tag{2}$$

where $I(R)$ is the surface brightness at projected radius R.

(b) The surface brightness distribution of many globular clusters is described by the Plummer law:

$$I(R) = \frac{I_0}{(1 + R^2/b^2)^2}. \tag{3}$$

Assuming that the cluster is spherically symmetric, use the result from (a) to calculate the escape velocity from the center of such globular cluster. Evaluate your result (in km/sec) for globular cluster M999 with central bolometric surface brightness $I_0 = 16$ mag/arcsec^2, core size $b = 3$ pc and mass-to-light ratio $\gamma = 2M_\odot/L_\odot$. (Hint: you need to translate mag/arcsec^2 into some more reasonable units first.)

(c) What is the luminosity density distribution for the surface brightness profile given by equation (3)?

(d) What is the total mass of the cluster M999, in solar masses?

5. Spherical potentials (2 points) (2.7 of BT) Astronauts orbiting an unexplored planet find that (i) the surface of the planet is precisely spherical and centered on $r = 0$; and (ii) the potential exterior to the planetary surface is $\Phi = -GM/r$ exactly, that is, there are no non-zero multipole moments other than the monopole. Can they conclude from these observations that the mass distribution in the interior of the planet is spherically symmetric? If not, give a simple example of a non-spherical mass distribution that would reproduce the observations.

6. Kolmogorov-Smirnov test (2 points). There are 200 datapoints in `http://zakamska.johnshopkins.edu/SPR2018/homework2.dat`. Plot the distribution (a pretty histogram) of these values. Is this distribution statistically consistent with a uniform distribution between 0 and 1? Explain what you are evaluating, provide some intermediate steps and explain your answer. (A similar question was on the questionnaire during the first lecture, and most people said that to evaluate whether a given set of values is drawn from some distribution one would use the χ^2 test – the χ^2 test is for evaluating whether the observed values $y_i(x_i)$ are consistent with values one would get from a model $f(x_i)$. To determine whether a set of values is drawn from some distribution, or whether two sets of values are drawn from the same distribution, you would use the KS test.)

7. Luminosity units (1 point). In astronomy, what are these the units of? (1) Janskys; (2) Watts; (3) mag/arcsec^2; (4) $L_\odot$; (5) mag; (6) Watts/m^2; (7) erg/sec. Which ones are units of luminosity? (This is also a question from the initial survey. Hint: nobody answered it correctly then, so think again.)