Homework 6 – AS.171.627 – Zakamska

0. Questions during talks (5 points). If at the time of grading you have logged two questions asked during the W&C talks or colloquia (other talks might qualify), you get five points.

1. Velocity anisotropy (3 points). [BT 4.6] The distribution function of a spherical system is proportional to $L^\gamma g(E)$. Show that the anisotropy parameter β is the same at all radii and equal to $-\gamma/2$. Is this a radially-biased or a tangentially-biased velocity distribution?

2. Black hole (4 points). A spherically symmetric globular cluster with an isotropic velocity dispersion tensor has a density profile

$$\rho(r) = \frac{\rho_0}{(1+r^2/b^2)^{5/2}}, \tag{1}$$

constant mass-to-light ratio, total mass $10^6 M_\odot$ and core radius $b = 5$ pc. Calculate the central line-of-sight velocity dispersion in km/sec. Suppose now there is a central black hole with $M_{BH} = 10^4 M_\odot$. The radius of influence r_{BH} of the black hole is defined as the distance at which the orbital velocity around the black hole equals the velocity dispersion of the stellar system. How big is the radius of influence, in pc? How much mass (in solar masses) is within the sphere of influence? What is the line-of-sight velocity dispersion in km/sec at $R = 0.5 r_{BH}$? Neglect the effects of two-body relaxation.

3. A two-component system (3 points). A spherically symmetric galaxy consists of stars and dark matter, and both components are important for producing the potential. Assume for simplicity that each component has isotropic velocity dispersion. Suppose stars have mass density and velocity dispersion $\rho_1(r)$, $\sigma_1(r)$ and dark matter particles have density and velocity dispersion $\rho_2(r)$, $\sigma_2(r)$.

(a) Write down the Jeans and Poisson equations for this system.

(b) Suppose that the ratio of stellar density to dark matter density is constant and equal to 0.2 (i.e., $\rho_1(r)/\rho_2(r) = 0.2$). What, if anything, can you say about the ratio of the velocity dispersions $\sigma_1(r)/\sigma_2(r)$?

4. X-ray atmosphere (3 points). A spherical galaxy is observed to be an extended source of X-rays with a surface brightness distribution given by

$$\Sigma_X(R) = \frac{\Sigma_0}{1+R/a}, \tag{2}$$

where a is the core radius and Σ_0 is the central surface brightness. The X-ray emission is assumed to be due to thermal bremsstrahlung of hot gas in hydrostratic equilibrium in the galaxy potential, and the gas itself contributes a negligible amount of mass. Aside from the overall normalization, the spectra of the X-ray emission seem to be independent of where in the galaxy they are extracted. Calculate the functional form of the galactic potential at large distances from the center, at $r \gg a$.

5. Hyper-velocity stars (3 points). What are Galactic hyper-velocity stars? What 2-3 processes generate hyper-velocity stars? Where in the Galaxy do they originate? Suppose you are asked to measure their proper motions with the ACS camera's Wide-Field Channel on the HST. How long would it take?

6. Dynamical distance (3 points). The stars of a spherically symmetric cluster have mean-square proper motions relative to the cluster mean $\langle \mu^2 \rangle$ and mean-square line-of-sight velocity relative to the mean $\langle v_{\|}^2 \rangle$. What is the cluster distance? Does this result depend on an assumption that the velocity ellipsoid is everywhere isotropic? This was the "dynamical distance" method used in Van de Ven et al. 2006 paper we discussed in class.