Homework 3 – AS.171.627 – Zakamska

1. Trajectory (1 point). For what spherically symmetric potential is a possible trajectory $r(\varphi) = a\exp(b\varphi)$? How can energy be conserved on such a rapidly expanding orbit?

2. Laplace equation (3 points). Show that $\Phi = V_c^2 \ln[r(1 + |\cos\theta|)]$ solves Laplace's equation everywhere except when $r = 0$ or $\theta = \pi/2$. Here r is the spherical radius and θ is the polar angle (defined to be 0 or π on the z axis). By applying Gauss's theorem near the equatorial plane $\theta = \pi/2$, find the surface density distribution in the equatorial plane that gives rise to this potential (what is this distribution called?).

3. Neutron star kick (3 points). A massive star is on a circular orbit in the mid-plane of the Milky Way at the solar Galactocentric radius $R_0 = 8.4$ kpc. It explodes as an off-center supernova (let's not worry about the plausibility of such scenario for now), which (in the frame co-moving with the initial orbit) gives the resulting neutron star a velocity kick $v_0 = 50$ km/sec straight away from the Galactic center. Assuming that the Milky Way potential is axisymmetric and using the epicyclic approximation, calculate the maximal fractional change of the angular momentum along this orbit. If we were to calculate this orbit exactly, without using the epicyclic approximation, what would be the maximal fractional change of the angular momentum?

4. Frequencies associated with the epicycles (1 point). Prove that at any point in an axisymmetric system at which the local density is negligible, the epicycle, vertical and circular frequencies κ, ν and Ω are related by $\kappa^2 + \nu^2 = 2\Omega^2$.

5. Plummer model (2 points). Calculate the gravitational potential distribution $\Phi(r)$ for the Plummer model from the previous homework. Plot the circular velocity profile in km/sec as a function of distance in pc for cluster M999.

6. Moments of a Kepler orbit (3 points). Compute the time-averaged moments of the radius, $\langle r^n \rangle$, in a Kepler orbit of semi-major axis a and eccentricity e, for $n = 1, 2$ and $n = -1, -2, -3$.

7. The Sun's orbit (3 points). Using the epicycle approximation, find the minimum and maximum distances from the Galactic center that the Sun attains in its orbit. You will need the values of the present solar distance from the Galactic center, the Oort's constants and the present solar velocity relative to the local standard of rest that we discussed in class.

8. Moving groups (2 points). The Hipparcos satellite has enabled us to determine accurate velocities for nearby stars, so that we can plot the density of nearby stars in velocity space (U, V, W velocity coordinate system defined in class) whose distance from the Sun is $\leq d_{\rm max}$. This distribution turns out to be lumpy (Figure 10.15 of Binney & Merrifield). These lumps are called moving groups and are commonly thought to consist of a set of stars that were born in a small region with very similar velocities, perhaps as part of a cluster that later dissolved.

In the epicycle approximation, a simple model of a moving group is a set of stars with the same guiding center radius R_g and the same radial epicycle amplitude X, but with a range of epicycle azimuthal phases; this model is intended to reflect the fact that the group spreads out in phase much more rapidly than it spreas out in guiding-center radius or epicycle amplitude. We will neglect motions perpendicular to the Galactic place and assume that $X \gg d_{\rm max}$. What would be the signature of such a group in the (U, V) plane?